

Difference between interaction cross sections and reaction cross sections

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We revisit the commonly accepted notion that the difference between interaction and reaction cross sections is negligible at relativistic energies, and show that, especially in small mass number region, it is large enough to help probe nuclear structure. For analyses of the difference, we construct “pseudo data” for the reaction cross sections using a phenomenological black-sphere model of nuclei since empirical data are very limited at high energies. The comparison with the empirical interaction cross sections suggests a significant difference between the reaction and interaction cross sections for *stable* projectiles on a carbon target, which is of the order of 0 – 100 mb.

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Measurements of interaction cross sections have been done for stable and light unstable nuclei [1] and are planned for heavy unstable nuclei in radioactive ion beam facilities, such as RIKEN RI Beam Factory. The interaction cross section, σ_I , for a nucleus incident on a target nucleus is defined as the total cross section for all processes associated with proton and/or neutron removal from the incident nucleus [2], which is measured by a transmission-type experiment. In this experiment, the cross section is obtained as

$$\sigma_I = (1/N_t) \log(\gamma_0/\gamma), \quad (1)$$

where γ is the ratio of the number of non-interacting nuclei to the number of incoming nuclei for a target-in run, γ_0 is the same ratio for an empty-target run, and N_t is the number of the target nuclei per cm² [2].

The above definition of σ_I leads to the relation, $\sigma_I = \sigma_R - \sigma_{\text{inel}}$, where σ_R is the total reaction cross section, and σ_{inel} is the cross section for inelastic channels as will be specified below. The total reaction cross section in turn satisfies the relation $\sigma_R = \sigma_T - \sigma_{\text{el}}$, where σ_T is the total cross section, and σ_{el} is the total elastic cross section.

In the measurements of σ_I , only the number of events in which an incoming nucleus has at least one nucleon removed is counted. The following processes are *not* counted in measuring σ_I : 1) Incident nuclei are excited without changing the original Z and N , and target nuclei can change in any way. 2) Target nuclei are excited without changing the original Z and N , while incoming nuclei remain in the ground state. 3) Target nuclei break up, while incoming nuclei remain in the ground state. These processes contribute exclusively to σ_{inel} and thus are included in σ_R . For incident nuclei with no excited bound states, such as ¹¹Li [3], we expect $\sigma_{\text{inel}} \cong 0$ and thus $\sigma_I \cong \sigma_R$.

In this Letter, we address the question of how large the difference between reaction and interaction cross sections is. We systematically analyze empirical data for the

reaction and interaction cross sections measured at high beam energy, $\gtrsim 800$ MeV, per nucleon. We complement the limited data for σ_R by constructing pseudo data with the help of a black-sphere picture of nuclei [4, 5].

Theoretically, Ogawa *et al.* pointed out that, for reactions of ¹¹Li with several kinds of targets, the contribution of σ_{inel} to σ_R is negligibly small [6]. Recently, Ozawa *et al.* experimentally estimated σ_{inel} for ³⁴Cl incident on a C target as less than about 10 mb [7]. Since $\sigma_I = 1334 \pm 28$ mb, the contribution of σ_{inel} to σ_R is also negligibly small. In both cases, however, the projectiles are loosely-bound systems. For stable nuclei, whether the contribution of σ_{inel} to σ_R is negligibly small or not is still an open question.

Recently, for the purpose of deducing nuclear size from proton-nucleus elastic scattering and reaction cross sections, we proposed a model in which a nucleus is viewed as a “black” sphere of radius a [4, 5]. Here we assume that the target nucleus is strongly absorptive to the incident proton and hence acts like a black sphere. Another requirement for the black-sphere picture is that the proton wave length is considerably smaller than the nuclear size. For proton incident energies higher than about 800 MeV, these requirements are basically satisfied.

In this scheme, we first evaluate the black-sphere radius, a , from the measured elastic diffraction peak and then identify a as a typical length scale characterizing the nuclear size [4]. The center-of-mass (c.m.) scattering angle for proton elastic scattering is generally given by $\theta_{\text{c.m.}} = 2 \sin^{-1}(q/2p)$ with the momentum transfer, \mathbf{q} , and the proton incident momentum in the c.m. frame, \mathbf{p} . For the proton diffraction by a circular black disk of radius a , we can calculate the value of $\theta_{\text{c.m.}}$ at the first peak as a function of a . (Here we define the zeroth peak as that whose angle corresponds to $\theta_{\text{c.m.}} = 0$.) We determine a in such a way that this value of $\theta_{\text{c.m.}}$ agrees with the first peak angle for the measured diffraction in proton-nucleus elastic scattering, θ_M . The radius, a , and

the angle, θ_M , are then related by

$$2pa \sin(\theta_M/2) = 5.1356 \dots \quad (2)$$

For scattering of protons having energies higher than ~ 800 MeV with stable nuclei, we obtained the following results [4, 5]: 1) the absorption cross section, πa^2 , agrees with the empirical total reaction cross section within error bars. Therefore, a can be regarded as a “reaction radius,” inside which the reaction with incident protons occurs. 2) $\sqrt{3/5}a (= r_{BS})$ almost completely agrees with the empirically deduced values of the root-mean-square matter radius for nuclei having mass $A \gtrsim 50$, while it systematically deviates from the deduced values for $A \lesssim 50$. We also found that, for stable nuclei ranging from He to Pb, the black-sphere radius scales as [5]

$$a \simeq 1.2135A^{1/3} [\text{fm}]. \quad (3)$$

From the scale a determined above, we calculate nucleus-nucleus absorption cross sections, which are to be compared with empirical total reaction cross sections, σ_R . We simply set

$$\sigma_{BS} = \pi(a_P + a_T)^2, \quad (4)$$

where $a_P(a_T)$ is the black-sphere radius of a projectile (target) nucleus. Here we assume that the incident protons are point particles as in Ref. [5]. By substituting the values of a_P and a_T determined by Eq. (2) into Eq. (4), we evaluate σ_{BS} for various sets of stable nuclei. Expression (4) is merely an assumption, but several available data support its validity as we will show later.

Now we concentrate on the reactions of stable projectile nuclei on a carbon target. Then, Eq. (4) reduces to

$$\sigma_{BS} = \pi(a_P + a(C))^2, \quad (5)$$

where $a(C)$ is the black-sphere radius of the target C nucleus obtained from the measured angle of the first diffraction maximum in proton elastic scattering [5]. For proton incident energy higher than ~ 800 MeV, $a(C) = 2.69 \pm 0.07$ fm. For later convenience, we introduce the interaction radius, a_I , through the following expression:

$$\sigma_I = \pi(a_I + a(C))^2. \quad (6)$$

In Fig. 1, we plot the empirical σ_R and σ_I data for incident energy per nucleon above ~ 800 MeV. For comparison, we also plot σ_{BS} . Since the number of the σ_R data is very limited in the energy region of interest here [10], we consider σ_{BS} as pseudo data for σ_R . σ_{BS} is useful for predicting σ_R for nuclides for which proton elastic scattering data are available while no data are available for σ_R . The dashed curve in the figure shows the scaling cross section, σ_{scaling} , for a nucleus- ^{12}C reaction, defined on the basis of Eq. (3) as

$$\sigma_{\text{scaling}} = \pi \left(1.2135A^{1/3} + a(C) \right)^2 [\text{fm}^2], \quad (7)$$

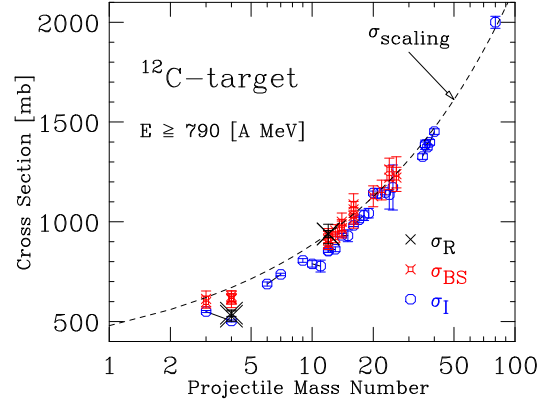


FIG. 1: (Color online) Comparison of the total reaction cross sections, σ_R (\times), and their substitutes σ_{BS} (squares) and σ_{scaling} (dashed curve) with the interaction cross sections, σ_I (\circ), for stable projectiles of $A \leq 80$ and a ^{12}C target. The absorption cross section, σ_{BS} , defined by Eq. (5) and the scaling cross section, σ_{scaling} , defined by Eq. (7) act as the pseudo data for σ_R . The empirical data for σ_R are taken from Ref. [8] (see text), and those for σ_I are taken from Refs. [1, 7, 9] for a projectile of $^3,^4\text{He}$, $^6,^7\text{Li}$, ^9Be , $^{10,11}\text{B}$, $^{12,13}\text{C}$, $^{14,15}\text{N}$, $^{16,17,18}\text{O}$, ^{19}F , $^{20,21}\text{Ne}$, ^{23}Na , $^{24,25}\text{Mg}$, $^{35,37}\text{Cl}$, $^{36,38,40}\text{Ar}$, and ^{80}Kr at energy per nucleon $\gtrsim 800$ MeV. This figure is the updated version of Fig. 4 of Ref. [5].

where $a(C)$ is fixed at 2.6930 fm. When the data for proton elastic scattering are not available, we adopt this σ_{scaling} as the pseudo data.

In fact, for incident energies per nucleon higher than ~ 800 MeV, only a few data are available for nucleus-carbon total reaction cross sections. For the σ_R data for $^{12}\text{C}+^{12}\text{C}$ at 870 MeV per nucleon, one finds 939 ± 17 mb and 939 ± 49 mb from Ref. [8]. By substituting these values into σ_{BS} in Eq. (4), one obtains $a_P = a_T = 2.73 \pm 0.03$ fm and 2.73 ± 0.07 fm, respectively. Note that this result is consistent with the value of $a(C)$ determined from proton elastic scattering data. For the σ_I data for the same reacting system, on the other hand, one finds 856 ± 9 mb at 790 MeV per nucleon and 853 ± 6 mb at 950 MeV per nucleon from Ref. [1]. From Eq. (6) one then obtains $a_I = 2.53 \pm 0.10$ fm and 2.52 ± 0.09 fm. The difference between $a(C)$ and this a_I is about 0 – 0.3 fm, which is typically of the order of neutron skin thickness for stable nuclei [11]. When we discuss the nuclear surface structure, therefore, such difference should be considered seriously.

As for the case of $^4\text{He}+^{12}\text{C}$, both σ_{BS} and σ_{scaling} significantly overestimate the empirical values of σ_R (542 ± 16 mb and 527 ± 26 mb) [8] and hence are not acceptable as the pseudo data for σ_R . This exceptional behavior is attributable to the fact that excitations associated with internucleon motion are highly suppressed in α particles [12].

One can see from Fig. 1 that σ_I is close to σ_{BS} in magnitude for the whole range of the projectile mass, but

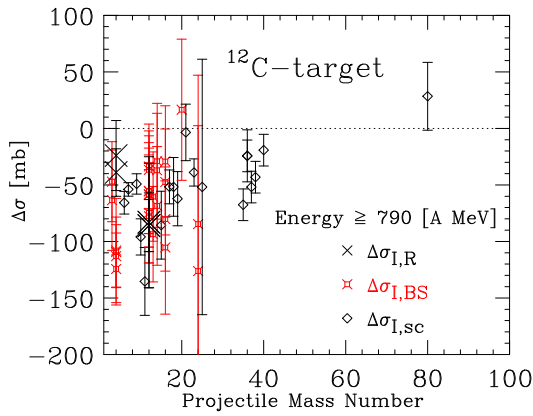


FIG. 2: (Color online) $\Delta\sigma_{I,R} = \sigma_I - \sigma_R$ (\times), $\Delta\sigma_{I,BS} = \sigma_I - \sigma_{BS}$ (squares), and $\Delta\sigma_{I,sc} = \sigma_I - \sigma_{scaling}$ (diamonds) as a function of projectile mass.

some deviations do exist. In Ref. [5], we simply stressed a good agreement of σ_{BS} with σ_I , while here we focus on the difference between these two.

In order to clarify the difference, we plot $\sigma_I - \sigma_{BS}$ or $\sigma_I - \sigma_{scaling}$, according to whether proton scattering data are available or not, for stable projectiles of $A \leq 80$ and a ^{12}C target in Fig. 2. From the figure, as expected, we find that the difference is mostly negative. The average of the difference over various projectiles is about -60.4 mb. This is the major finding of this Letter. Interestingly, the difference seems to decrease in magnitude with the projectile mass A although the plotted data are rather sparse and often accompanied by large error bars. We claim that the difference due to the choice of σ_{BS} or $\sigma_{scaling}$ instead of σ_R is not any artifact. The validity of this choice will be discussed later.

Let us proceed to analyze the A -dependence of $\sigma_I - \sigma_{BS}$ under the assumption that $\sigma_R = \sigma_{BS}$. Using Eqs. (5) and (6), we can express the difference as

$$\sigma_I - \sigma_{BS} \simeq -2\pi (a_P + a(C)) \Delta a, \quad (8)$$

where $\Delta a \equiv a_P - a_I$. The above assumption ensures $\Delta a > 0$. If Δa were independent of A , $|\sigma_I - \sigma_{BS}|$ would grow like $A^{1/3}$ as a_P behaves as in Eq. (3). As Fig. 2 suggests, however, $|\sigma_I - \sigma_R|$ approaches a vanishingly small value or at least does not increase with A . This implies that Δa decreases no slower than $A^{-1/3}$ as A increases. As we shall see, this A -dependence has implications for nuclear structure, a feature at odds with the commonly accepted notion that the difference between interaction and reaction cross sections is negligible at relativistic energies.

Then, what is the physical implication of the difference, Δa ? From the aforementioned interpretation of the black-sphere radius as a reaction radius for incident protons and the fact that the np total cross section is similar to the pp total cross section at high incident energy above ~ 800 MeV, we may assume that a_P corresponds

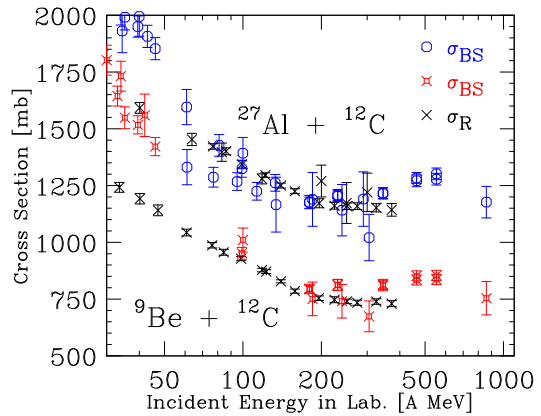


FIG. 3: (Color online) Comparison of σ_{BS} (squares for $^9\text{Be} + ^{12}\text{C}$ and circles for $^{27}\text{Al} + ^{12}\text{C}$) with σ_R (\times) for the reactions of $^9\text{Be} + ^{12}\text{C}$ (lower) and $^{27}\text{Al} + ^{12}\text{C}$ (upper) as a function of incident energy per nucleon. The data of σ_R are taken from Refs. [10, 15, 16].

to a critical radius of a projectile nucleus inside which reactions occur with nucleons in a target C nucleus. Note that a_P is located in the surface region. At a radius of a_I , which is only slightly smaller than a_P , transfer of incident energy into excitations of nucleons inside the projectile nucleus has to be more effective than at a radius of a_P because of more frequent reactions and, eventually, enough to induce nucleon emission. We thus expect that Δa has relevance to the energy scale characterizing breakup of the projectile nucleus, such as single-particle level spacing and separation energies. In fact, the A -dependence of Δa mentioned above could be a key to clarifying what energy scale controls nucleon emission.

We remark that the present discussion is not always applicable when projectiles are deformed nuclei [13]. In this case, σ_I could be appreciably smaller than σ_R even if A is relatively large. This is because a significant part of σ_{inel} ($= \sigma_R - \sigma_I$) comes from the low-lying rotational excitations of the projectile nucleus. Candidates for heavy stable nuclei that are deformed in the ground state are ^{80}Kr , ^{154}Sm , ^{176}Yb , etc., but at least for ^{80}Kr , the effect is not seen as long as we assume $\sigma_R = \sigma_{scaling}$.

One may wonder if our arguments based on the assumption, $\sigma_R \simeq \sigma_{BS}$, and Eq. (4) are valid because of a severe shortage of the real σ_R data at incident energy above ~ 800 MeV per nucleon. Even for the available data for $^{12}\text{C} + ^{12}\text{C}$ at 870 MeV per nucleon [8], which we have to rely heavily on, its validity remains to be checked.

In order to lessen such a concern, we proceed to show that σ_{BS} given by Eq. (5) does work as a substitute for σ_R . On the basis of the fact that σ_R for proton-nucleus reactions agrees with πa^2 within error bars [5] and that this tendency persists for proton incident energy down to about 100 MeV [14], we rederive a_P and $a(C)$ from the corresponding σ_R [17, 18] as $(\sigma_R/\pi)^{1/2}$, rather than from proton elastic scattering data. (Note that one cannot

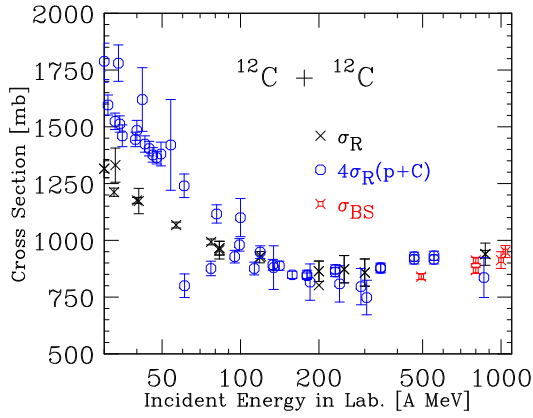


FIG. 4: (Color online) Comparison of $4\sigma_R(p+C)$ (\circ) with σ_R for $^{12}\text{C}+^{12}\text{C}$ reactions (\times) as a function of incident energy per nucleon. The $\sigma_R(p+C)$ data are taken from Refs. [17, 18], while the σ_R data for $^{12}\text{C}+^{12}\text{C}$ reactions are taken from Refs. [8, 10, 15, 16, 19, 20, 21, 22, 23, 24]. For reference, we also plot $\sigma_{BS} = 4\pi a(C)^2$, where $a(C)$ is determined from the measured angle of the first diffraction peak of proton elastic scattering, by squares.

determine a_P for C and lighter projectile nuclei and $a(C)$ from elastic scattering angular distributions measured for the proton incident energies less than ~ 400 MeV, which lack the peak structure.) When calculating a_P and $a(C)$ to obtain σ_{BS} for projectile-carbon reactions at a given incident energy per nucleon, T_P , we adopt the values of σ_R measured at proton incident energies within ~ 5 MeV of T_P . The obtained values of σ_{BS} are to be compared with the σ_R data taken with a ^{12}C target, which are, at high incident energy, presently limited to such projectiles as ^9Be , ^{12}C , and ^{27}Al [10, 15, 16].

In Fig. 3, we plot σ_{BS} and σ_R for the reactions of $^9\text{Be}+^{12}\text{C}$ and $^{27}\text{Al}+^{12}\text{C}$. The agreement between σ_{BS} and σ_R is fairly good for incident energies per nucleon ranging $\sim 100 - 400$ MeV. Although the uncertainties in σ_{BS} are still large, due mainly to the uncertainties in the measured values of proton-nucleus total reaction cross sections, such agreement strongly supports the effectiveness of Eq. (5) at predicting σ_R for energies per nucleon higher than ~ 100 MeV. Apparently, the corresponding ratio of σ_{BS} to σ_R fluctuates within $\sim 10\%$ of unity, but if we restrict ourselves to the recent data [16, 18], the fluctuation becomes much smaller.

Another example is σ_R versus σ_{BS} for $^{12}\text{C}+^{12}\text{C}$ reactions. In the black-sphere approximation, σ_{BS} for $^{12}\text{C}+^{12}\text{C}$ reactions is expressed as $\sigma_{BS} = 4\pi a(C)^2$, because in this case $a_P = a(C)$ in Eq. (5). Let us assume that $\pi a(C)^2$ is equal to $\sigma_R(p+C)$, where $\sigma_R(p+C)$ is the empirical total reaction cross section for protons incident on a carbon target [17, 18]. Then, if σ_R for $^{12}\text{C}+^{12}\text{C}$ reactions is equal to $4\sigma_R(p+C)$, we can show that Eq. (5) works also for the case of $^{12}\text{C}+^{12}\text{C}$ reactions.

In Fig. 4, we compare $4\sigma_R(p+C)$ with σ_R for $^{12}\text{C}+^{12}\text{C}$

reactions as a function of incident energy per nucleon. For incident energies per nucleon higher than ~ 100 MeV, we obtain an excellent agreement between them. This implies the validity of the black-sphere picture based on the empirical relation $\sigma_R \cong \sigma_{BS}$ for these energies.

Note that $4\sigma_R(p+C)$ is appreciably larger than σ_R for $^{12}\text{C}+^{12}\text{C}$ reactions for incident energies per nucleon less than about 100 MeV. This implies that the classical picture, the geometrical description of the cross section underlying Eq. (5), breaks down. If we could adopt $4\sigma_R(p+C)$ as a basis in assessing σ_R for $^{12}\text{C}+^{12}\text{C}$ reactions, we need a certain “transparency” effect [15] to reproduce the data. We remark that the Coulomb effect on σ_R would be hard to resolve the disagreement alone, because the cross-section reduction due to the Coulomb repulsion between the projectile and target nuclei is stronger for proton-carbon reactions than for carbon-carbon ones at given incident energy per nucleon. Similar deviations also appear for $^9\text{Be}+^{12}\text{C}$ and $^{27}\text{Al}+^{12}\text{C}$ cases of incident energy per nucleon lower than about 100 MeV as in Fig. 3.

We should be careful about a deviation of σ_{BS} from $4\sigma_R(p+C)$ that is appreciable around 500 MeV per nucleon in Fig. 4. It does not imply a flaw in the black-sphere model, but simply reflects the fact that the measured values of $\sigma_R(p+C)$ at proton incident energies of 220–570 MeV [25] are larger than expected from the systematics [12]. Note that a similar tendency appears in the values of σ_{BS} for $^9\text{Be}+^{12}\text{C}$ and $^{27}\text{Al}+^{12}\text{C}$ reactions derived from the same values of $\sigma_R(p+C)$ (see Fig. 3).

In summary, we have pointed out that, for stable nuclei incident on a carbon target, there is a significant difference between real σ_I data and pseudo σ_R data even at relativistic energies, especially at small mass number. This difference would lead to possible uncertainties of about 0–0.3 fm in estimates of nuclear matter radii, if relying on the σ_I data alone. We have found that this difference is consistent with the fact that $\sigma_I < \sigma_R$ and generally stays within 0–100 mb. The difference is clear for small A while it is less clear for larger A . This implies that the scale Δa characterizing the difference between the black-sphere and interaction radii of the projectile nucleus decreases no slower than $A^{-1/3}$ as A increases, a feature relevant to the problem of what energy scale controls the breakup of the projectile. Of course, the above implications strongly depend on the validity of our black-sphere picture, which is based on $\sigma_R \cong \sigma_{BS}$. The presently available σ_R data for ^9Be , ^{12}C , and ^{27}Al incident on ^{12}C support the above relation.

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